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Simplex sliding mode control for nonlinear uncertain systems via chaos optimization

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Abstract

As an emerging effective approach to nonlinear robust control, simplex sliding mode control demonstrates some attractive features not possessed by the conventional sliding mode control method, from both theoretical and practical points of view. However, no systematic approach is currently available for computing the simplex control vectors in nonlinear sliding mode control. In this paper, chaos-based optimization is exploited so as to develop a systematic approach to seeking the simplex control vectors; particularly, the flexibility of simplex control is enhanced by making the simplex control vectors dependent on the Euclidean norm of the sliding vector rather than being constant, which result in both reduction of the chattering and speedup of the convergence. Computer simulation on a nonlinear uncertain system is given to illustrate the effectiveness of the proposed control method.

1. Introduction

As a powerful tool for solving complex control problems under significant uncertainties, sliding mode control (SMC) has been widely studied and applied in different fields of engineering applications over the last 30 years. The primary feature of sliding-mode control systems is that the feedback signal is discontinuous, switching on one or more manifolds in the state space. When the system state crosses each discontinuity surface, the structure of the feedback system is altered. Under certain circumstances, all motions in the neighborhood of the desired manifold are directed towards the manifold and thus a sliding motion on a predefined subspace within the state space is established, in which the system state repeatedly crosses the switching surface. During the sliding mode, the system possesses high robustness against uncertainties of various kinds, such as parameter variations and external disturbances.

However, due to the use of discontinuous control signals, the sliding mode control inherently suffers from the chattering problem. For an m-input system, the conventional sliding-mode control approach usually partitions the state space into 2^m regions and the control law will switch whenever the system state moves from one region to another. As a consequence, unwanted high-frequency chattering motions are generated, which degrades the system performance in general. To reduce the switching behaviour and circumvent the redundancy of control actions that occurs in the case of the conventional component-wise sliding-mode control design procedure, Baida and Izosimov [1] proposed a new algorithm, called the simplex sliding mode control (SSMC) scheme, which only partitions the state space into $m+1$ regions. Clearly, the number of regions is decreased and as a result the chattering problem can be improved. Besides,

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SSMC is also an effective way to extend the sliding-mode control methodology to the multi-input case. The design of the so-called simplex control for multivariable systems [2,3] is relatively straightforward in comparison with the conventional SMC, and some successful applications of SSMC in practice have been reported [4].

SSMC differs in certain fundamental aspects from the conventional SMC. For instance, the state space is partitioned into only $m+1$ regions, instead of 2^m regions as is typical for conventional SMC, where m is the number of control inputs. Furthermore, a distinct control vector is used in each region of the partition for the simplex control scheme, and these $m+1$ control vectors satisfy the simplex property (to be defined later) in a reduced-order subspace whose origin corresponds to the intersection of the surfaces $S_ix = 0$, $i = 1, \dots, m$, which is the desired sliding mode hypersurface in the state space. In addition, for this scheme (except for $m = 1$) the surfaces of control discontinuity do not coincide with the linear sliding hypersurfaces $S_ix = 0$, $i = 1, \dots, m$, as is the case for conventional SMC; instead, there exist $m(m+1)/2$ switching hypersurfaces for the control.

In other respects, the SSMC design methodology is similar, in principle, to the conventional SMC design. Having first selected $S = [S_1^T \quad S_2^T \quad \cdots \quad S_m^T]^T$ to obtain some desired sliding-mode dynamics on the hypersurface $Sx = 0$ (sliding phase design), choose next the control vectors so that the closed-loop system trajectory will be forced into a sliding mode on that hypersurface (reaching phase design).

The simplicity and applicability of SSMC are so appealing that there have been some publications about how to synthesize the SSMC law for linear time-invariant systems [5,6]; however, no systematic approach is currently available for computing the simplex control vectors in nonlinear sliding mode control. In this paper, an innovative global optimization method, chaos optimization, is exploited to develop a systematic approach to seeking the simplex control vectors for nonlinear sliding mode control systems, which significantly extends the state-of-the-art advances of the methodology.

The rest of the paper is organized as follows. In the next section, the simplex sliding mode control and the chaos optimization are introduced. Section 3 presents the procedure for synthesizing the simplex sliding mode control law via the chaos optimization. In Section 4, the proposed synthesis approach is simulated on an uncertain multi-input nonlinear system to confirm its validity. Finally, some concluding remarks are given in Section 5.

2. Preliminaries

2.1. Simplex-type nonlinear sliding-mode control

Considering a class of affine nonlinear systems:

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

where $x \in R^n$, $u \in R^m$. Choose a manifold described by $s(x) = Hx = 0$, where $s(x) \in R^m$, so that if the system trajectories lie on the manifold, the behaviour of the system satisfies a prespecified control objective. Assume that a control signal is designed so that it is capable of forcing the system trajectories from any initial state to reach the manifold in finite time and then to slide on it.

Sliding-mode control (SMC) in the single-input case is quite straightforward and has been well developed. In contrast, most methods of designing multi-input SMC for nonlinear uncertain systems are far from a simple generalization of the standard single-input variable structure approach.

In order to provide a control strategy accomplishing the objective of steering $s(x)$ to zero in the multi-input case, consider the dynamic behaviour of the m-vector $s(x)$ associated with system (1), namely,

$$\dot{s}(x) = Hf(x) + Hg(x)u, \tag{2}$$

where the matrix $[Hg(x)]$ is assumed to be always invertible.

A traditional way of extending the basic SMC strategy to the multi-input case consists of selecting the control signals according to a component-wise procedure. This means that the control is designed so that each component of the vector $s(x)$ is related to the corresponding time derivative $\dot{s}(x)$ by the so-called reaching condition. Consider for instance the constant plus proportional rate reaching law [7]:

$$\dot{s} = -Q\,\mathrm{sign}(s) - Ks, \tag{3}$$

where $Q = \mathrm{diag}\{q_i^2\}$, $K = \mathrm{diag}\{k_i^2\}$, $i = 1, \dots, m$ function as the weighting matrices. The control law derived from (2) and (3) becomes

$$u = -[Hg(x)]^{-1}[Hf(x) + Q\operatorname{sign}(s) + Ks]. \tag{4}$$

It is noteworthy that each component of the vector u is a linear combination of some discontinuous signals.

An alternative approach to multi-input nonlinear sliding-mode control is the simplex control [2,3]. The advantage of the simplex control approach lies in the fact that the number of structures among which the controlled system switches is reduced with respect to the component-wise case ($m+1$ instead of 2^m). To introduce this method, the following definitions are needed.

Definition 1. The set $U = \{u^1, u^2, \ldots, u^{m+1}\}$, where $u^i \in R^m$, $i = 1, 2, \ldots, m+1$, are distinct and nonzero vectors, is said to form a simplex in R^m if every subset of m vectors in U are linearly independent and there exist $m+1$ real and positive constants $\{\alpha_1, \alpha_2, \ldots, \alpha_{m+1}\}$ such that

$$\sum_{i=1}^{m+1} \alpha_i = 1 \quad \text{and} \quad \sum_{i=1}^{m+1} \alpha_i u^i = 0. \tag{5}$$

This definition means that a simplex is a set of $m+1$ affinely independent vectors in R^m such that 0^m is in the interior of the convex hull of those vectors. For simplex-type nonlinear sliding-mode control, the following transformation is performed:

$$\sigma = [Hg(x)]^{-1} s(x). \tag{6}$$

Next, define the $m+1$ open regions in the m-dimensional σ-space associated with such a simplex as

$$\Omega_i = \left\{ \sigma : \sigma = \sum_{j=1, j\neq i}^{m+1} \lambda_j u^j, \lambda_j > 0 \right\}, \quad i = 1, 2, \ldots, m+1. \tag{7}$$

Thus, each Ω_i is an infinite cone, with vertex at 0^m, situated on the side of R^m opposite to u^i.

Definition 2. Suppose that the set $U = \{u^1, u^2, \ldots, u^{m+1}\}$ forms a simplex in the m-dimensional σ-space. The simplex control law is defined to be

$$u = u(\sigma) = u^i, \quad \text{for } \sigma \in \Omega_i. \tag{8}$$

Note that given a simplex U, the control switching surfaces for the simplex control law are defined by the boundaries of Ω_i, which is apparently not accordant with the sliding manifold. The following theorem [2,3] investigates the stability and convergence of the simplex nonlinear sliding mode control approach.

Theorem 1. *Partition the σ-state space into $m+1$ non-overlapping regions as* (7). *If the control is defined by* (8), *then the origin $\sigma = 0$ is asymptotically stable and the convergence takes place in finite time.*

From (6), it is clear that the above theorem guarantees the closed-loop system defined by (1) and (8) will achieve the sliding mode in finite time, i.e., $s(x) = Hx = 0$. Note that the control is unique in each region, and each region is uniquely determined by the choice of the simplex of control vectors.

2.2. Chaos optimization

Over the last decade, one has been experienced a resurgence of interest in the control systems society for new methods of global optimization as well as the application of available global optimization algorithm to control systems analysis and synthesis. This recent surge of interest is attributed to the fact that a number of problems in the field of robust control may be directly restated as an optimization problem.

Genetic algorithm (GA) and simulated annealing (SA) are among the most popular tools for global optimization [8,9]. By mimicking the metaphor of natural biological evolution, GA operates on a population of potential solutions applying the principle of survival of the fittest to produce successively better approximations to a solution. Inspired by the annealing process for metals during cooling, SA describes a family of iterative methods where every iteration consists of taking a step in the parameter space with a probability that decays exponentially with an "energy" function associated with the new parameter value.

As derivative-free stochastic search methods for determining the solutions of an optimization problem, simulated annealing and evolutionary computation both employ a stochastic mechanism to avoid being trapped in a local optimum. On the contrary, the deterministic dynamics plays a central role in the method of chaos optimization [10].

Chaos, an apparently disordered behavior that is nonetheless deterministic, is a universal phenomenon that occurs in many nonlinear systems. It is featured by highly unstable motion of deterministic systems in a bounded region of the phase space. High instability means that the distance of two nearby orbits increases exponentially with time [11], which is a result of the extreme sensitivity of chaotic systems to the initial conditions. The Lyapunov exponents quantify this property. The magnitude of a Lyapunov exponent represents the principal rate of orbit divergence in the phase space. For a one-dimensional dynamical system $x_{i+1} = f(x_i)$, the Lyapunov exponent λ is defined as the long-time average of the exponent with respect to an orbit:

$$\lambda = \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N-1} \log |f'(x_i)|. \tag{9}$$

Chaos is then characterized by the boundedness of the system trajectories with a positive Lyapunov exponent, which implies that the average gradient of the map is greater than unity, and accordingly two nearby orbits in phase space diverge at an exponential rate.

It was emphasized that the sensitivity to the initial value suggests the irregularity of the series $\{x_i\}$ generated by chaos [11]. Consider the ith number x_i of the series. It may be possible that x_j with $j > i$ is quite close to x_i. Unless $x_i = x_j$ exactly, however, the part $x_i, x_{i+1}, x_{i+2}, \ldots$ is very different from the part $x_j, x_{j+1}, x_{j+2}, \ldots$ due to the sensitivity to the initial difference.

Although the long-term behaviour of a chaotic system shows typical stochastic properties, chaos is not equivalent to a random process. A chaotic motion can traverse every state in a certain region (called the chaos space) by its own regularity, and every state is visited only once therefore no precise periodicity. The unique ergodicity and the irregularity of the series generated by chaos make chaotic dynamics a potential candidate in the field of global optimization [10]. In fact, it has been successfully applied in solving nonlinear programming problems [12] and improving the performance of the GA [13].

In our study, the *logistic* mapping is used in chaos optimization to generate the chaotic time series. Consider the equation of *logistic* mapping

$$x_{i+1} = 4x_i(1 - x_i), \tag{10}$$

where x_i is the chaotic variable. Its Lyapunov exponent is

$$\lambda = \log 2 > 0. \tag{11}$$

It is well known that chaotic evolutions could be generated by Eq. (10), and the ergodic area (i.e., chaos space) is the interval $(0, 1)$. A general procedure of chaos optimization can be found in [10].

3. Synthesis of the simplex control law via chaos optimization

The simplex sliding mode control is based on a set of $m + 1$ control vectors forming a simplex in R^m, and on the corresponding switching of the controlled system from one to another among $m + 1$ different structures. In other words, this approach consists of the selection of a set of control vectors forming a simplex, along with a suitable switching logic. Hence, the crux of simplex control is the choice of a proper set of simplex control vectors. Once they are appropriately decided, the system state space will be partitioned into $m + 1$ non-overlapping regions, and with each region a particular control vector, among those of the simplex, is associated in such a way that the system trajectory is forced to slide on a prespecified manifold.

In our study, the simplex control vectors were sought via chaos optimization. Contrary to the use of constant simplex in the previous works, the simplex control vectors employed here is dependent on the Euclidean norm of the sliding vector so that the control effort is proportional to the energy of the sliding vector, which results in reduction of the chattering and speedup of the convergence. Clearly, the simplex control vectors include a total of $m \times (m + 1)$ entries, where m is the number of system inputs.

It turns out to be much easier to describe the proposed method by working out a concrete example in detail, rather than using some general statements with complicated formulas. For this purpose, consider the following uncertain nonlinear system:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = (c-a)x_1 - x_1x_3 + cx_2, \\ \dot{x}_3 = x_1x_2 - bx_3, \end{cases} \tag{12}$$

where $a = 45$, $b \in [3.5, 11]$, $c \in [25, 31]$. This nonlinear model is based on the chaotic Chen's system [14], and it follows from its bifurcation analysis [15] that this interval system remains to be chaotic when c is fixed to be 28. It has been widely experienced that this chaotic system is relatively difficult to control as compared to the Lorenz system and Chua's system, which are all topologically non-equivalent, due to the prominent three-dimensional and complex topological features of its attractor, especially its rapid change in velocity in the x_3-direction.

The interval system (12) can be represented by a simple state-space equation of the form

$$\dot{x}(t) = f_0(x) + \Delta f(x), \quad x(t) \in R^3, \tag{13}$$

where $\dot{x} = f_0(x)$ the nominal system, which can be described by

$$\begin{cases} \dot{x}_1 = a_0(x_2 - x_1), \\ \dot{x}_2 = (c_0 - a_0)x_1 - x_1x_3 + c_0x_2, \\ \dot{x}_3 = x_1x_2 - b_0x_3, \end{cases} \tag{14}$$

where $a_0 = 45$, $b_0 = 7.25$, $c_0 = 28$ and $\Delta f(x)$ denotes the system perturbations. The controlled uncertain nonlinear system is

$$\dot{x}(t) = f_0(x) + \Delta f(x) + Bu(t), \tag{15}$$

where $u(t) \in R^2$ and

$$B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Given $b = b_0 + \Delta b$ and $c = c_0 + \Delta c$, the uncertainties can be decomposed into

$$\Delta f(x) = \begin{pmatrix} 0 \\ \Delta c \cdot (x_1 + x_2) \\ -\Delta b \cdot x_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \Delta c \cdot (x_1 + x_2) \\ -\Delta b \cdot x_3 \end{pmatrix} = B \cdot \Delta \tilde{f}(x). \tag{16}$$

Obviously, the uncertainties $\Delta f(x)$ in system (15) satisfy the *matching conditions*, namely, $\Delta f(x)$ is in the subspace spanned by the column vectors of B. So, the system in sliding mode is robust to the system perturbations $\Delta f(x)$.

In order to find a sliding mode control law, $u(t) \in R^2$, which can guide the system state $x(t)$ to track the prespecified reference signal $x_r(t)$, a sliding surface needs to be defined for the nominal system beforehand. The desired sliding manifold can be obtained by using the method proposed in [16]:

$$S(e) = \{e | s(e,t) = He(t) = 0\}, \quad s(e) \in R^2, \tag{17}$$

where $e = x - x_r$ is the tracking error vector for the desired trajectory $x_r(t)$.

Then, one can start the procedure for synthesizing the simplex control vectors. Firstly, the performance index was defined as

$$\Gamma(U) = \sum_{i=1}^{N} \{\|s_i\| + \mu\|\Delta s_i\|\}, \tag{18}$$

where N is the duration of the simulation for evaluating the design, i the time index in simulation, s_i the sliding vector at simulation step i, Δs_i the change of the sliding vector and μ a bias weighting factor between s_i and Δs_i. The Δs_i term can be distinctively weighted to further suppress oscillations. The simplex control vectors will be decided to minimize the performance index (18), where $\|\cdot\|$ denotes the Euclidean norm of a vector.

Since six components are involved in the simplex control vectors $U = \{u^1, u^2, u^3\}$, six initial chaotic variables, $\gamma_{1,0}, \gamma_{2,0}, \ldots, \gamma_{6,0}$, $0 \leqslant \gamma_{j,0} \leqslant 1$, $j = 1, 2, \ldots, 6$, are selected randomly, and the fixed points 0.25, 0.5, 0.75 of the *logistic* map cannot be used as initial variables. The lower bounds and upper bounds of the searched variables are denoted as lbd_j and ubd_j, $j = 1, 2, \ldots, 6$, and U^* and Γ^* are assumed to be the optima. The search procedure by chaos optimization starts from $n = 0$:

Step 1: Chaotify the variables:
Substitute $\gamma_{1,n}, \gamma_{2,n}, \ldots, \gamma_{6,n}$ the equation of (10) to generate six chaotic variables $\gamma_{1,n+1}, \gamma_{2,n+1}, \ldots, \gamma_{6,n+1}$ via the *logistic* map.

Step 2: Perform the transformation from the chaotic space to the solution space by using the following formula:

$$\beta_{j,n+1} = \mathrm{lbd}_j + (\mathrm{ubd}_j - \mathrm{lbd}_j) \times \gamma_{j,n+1}, \quad j = 1, 2, \ldots, 6. \tag{19}$$

Step 3: Determine if $K = \{\kappa^1, \kappa^2, \kappa^3\}$, where $\kappa^i = [\beta_{i\times 2-1,n+1},\ \beta_{i\times 2,n+1}]^{\mathrm{T}}$, from a convex hull and the origin of σ-space is enclosed by the convex hull, i.e., $O^2 \in \mathrm{co}\{\kappa^1, \kappa^2, \kappa^3\}$; if not, then let $n = n + 1$ and return to step 1; if yes, then $K = \{\kappa^1, \kappa^2, \kappa^3\}$ has formed a simplex.

Step 4: Compute the sliding vector $s = He(t)$ and $\sigma = [HB]^{-1}s$, and then assign the simplex $U = \{u^1, u^2, u^3\}$ where $u^i = \kappa^i \cdot \|s\|$, to be the control vectors.

Step 5: Decide to which open region, partitioned by the simplex control vectors $U = \{u^1, u^2, u^3\}$, the σ belongs. In light of the definition given by (7), this problem can be solved by minimizing the following objective function:

$$E_i = \left\| \sigma - \sum_{j=1, j\neq i}^{m+1} \lambda_j u^j \right\|, \quad i = 1, 2, \ldots, m + 1, \tag{20}$$

subject to the constraints of $\lambda_j > 0$. The MATLAB command *Isqnonneg* can be used to resolve this plain linear programming problem in simulation. If there exist m coefficients $\lambda_j > 0$ such that $E_i \to 0$, then $\sigma \in \Omega_i$.

Step 6: Apply the simplex control law (8) for tracking control of the nominal system (14) in the duration of the simulation.

Step 7: Compute the performance index Γ_{n+1} in (18), and assign the optima as follows:

$$\text{If } (n = 0) \text{ or } (\Gamma_{n+1} \leqslant \Gamma^*) \quad \text{then } \Gamma^* = \Gamma_{n+1},\ U^* = U_{n+1} \text{ else do nothing;}$$

$n = n + 1$, and then return to step 1.

Repeating the above procedure, in finite time, one can get the optima Γ^* and U^*, which minimize the performance objective (18).

4. Simulation study

In the simulation, the objective is to use two control inputs, $u(t) \in R^2$, to guide the state $x(t) \in R^3$ of system (1) to match a prespecified reference signal $x_r(t)$. Note that this is much more difficult than the tasks achieved before [17,18].

The reference trajectory $x_r(t)$ is specified as a closed orbit corresponding to a periodic solution of the unforced Chen's equation. Let the parameters of system (12) be $a = 45$, $b = 1.5$ and $c = 28$, so that system (12) generates a periodic solution [15]. Starting from the initial state $x_r(0) = [-1.7570, -1.9648, 7.9743]^{\mathrm{T}}$, the three-dimensional phase portrait and the time-domain response of the reference $x_r(t)$ are shown in Figs. 1 and 2, respectively.

For the purpose of controlling the nonlinear uncertain systems (1) by sliding mode control, the desired sliding manifold was firstly constructed via the method proposed in [16]:

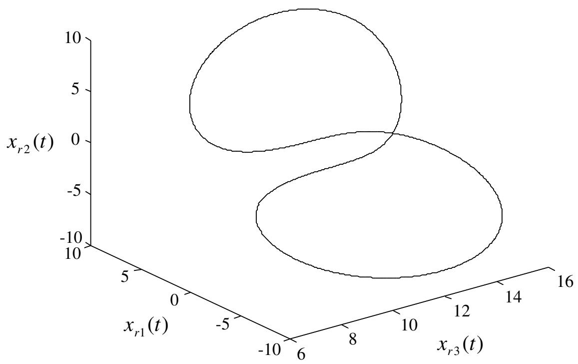

Fig. 1. The reference orbit $x_r(t)$, plotted in the $x_3 - x_1 - x_2$ space.

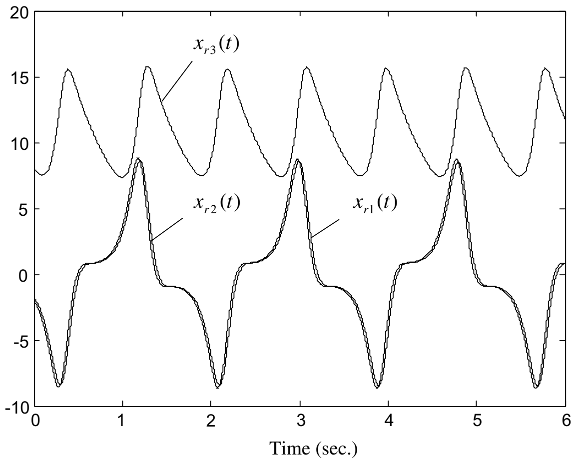

Fig. 2. The deterministic time series of the reference orbit $x_r(t)$.

$$S(e) = \{e|s(e,t) = He(t) = 0\}, \quad s(e) \in R^2,$$

where $e = x - x_r$ and

$$H = \begin{bmatrix} 4.1680 & 7.1120 & 9.6191 \\ 7.4080 & 9.7120 & -3.4480 \end{bmatrix}. \tag{21}$$

Next, in light of the procedure presented in Section 3, the optimal simplex control vectors were sought via chaos optimization. The upper bound and lower bound of the searched variables are chosen as $\mathrm{lbd}_j = -50$ and $\mathrm{ubd}_j = 50$, and the procedure was performed for about 300 times. The result is $U^* = \{u^1, u^2, u^3\}$, $u^i = \kappa^i \cdot \|s\|$ and

$$\begin{aligned} \kappa^1 &= [0.0778, 47.1185]^T, \\ \kappa^2 &= [-49.992, -10.0773]^T, \\ \kappa^3 &= [26.2508, -3.5633]^T. \end{aligned} \tag{22}$$

With the obtained simplex control vectors U^*, the tracking control performance under the simplex sliding mode control law (8) is visualized by Figs. 3–6, which shows that the trajectory of the controlled chaotic system is steered to the reference signal with satisfactory performance.

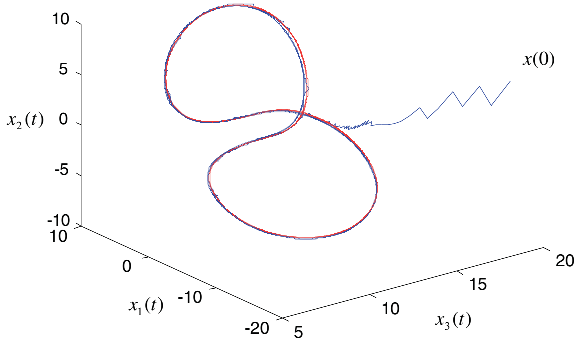

Fig. 3. The controlled trajectory $x(t)$ of Chen's chaotic system to the reference orbit $x_r(t)$, plotted on the $x_3 - x_1 - x_2$ space.

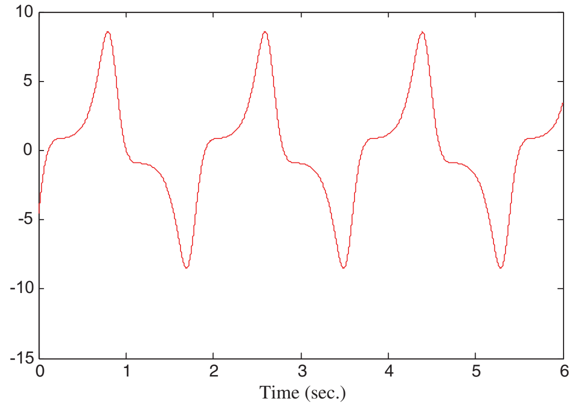

Fig. 4. Tracking performance of the first coordinate.

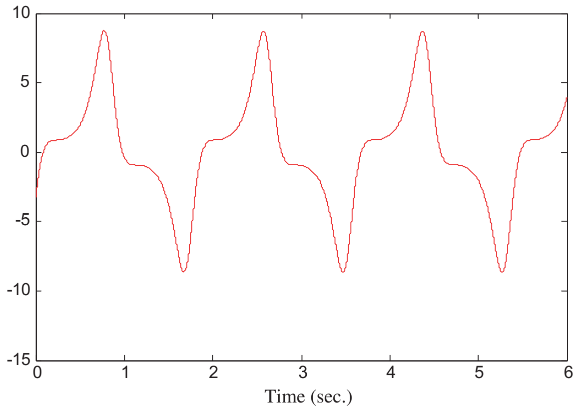

Fig. 5. Tracking performance of the second coordinate.

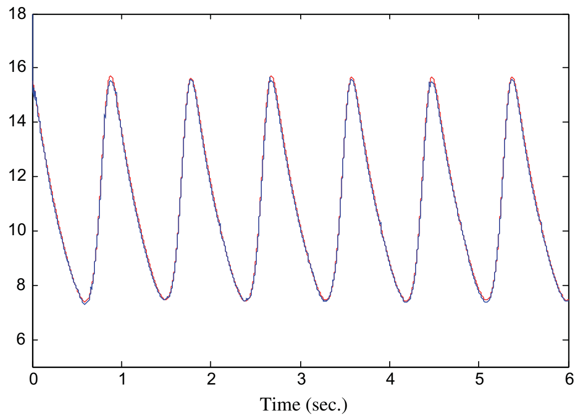

Fig. 6. Tracking performance of the third coordinate.

5. Conclusions

It is known that the simplex sliding mode control has made several important improvements over the conventional sliding mode control for robust control of nonlinear dynamical systems, and yet the selection of the magnitudes and directions of the control vectors in this method is usually determined by trial-and-error through simulation and/or experimental testing. In this paper, a systematic controller design method based on chaos optimization has been developed for simplex sliding mode tracking control for affine-type of uncertain nonlinear systems with multi-inputs. This method provides a step-by-step routine for constructing the simplex control vectors for the nonlinear sliding mode control, thereby resolving a long-lasting difficult technical problem in robust control. Computer simulation on a nonlinear uncertain system has illustrated the effectiveness of the proposed design method.

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